Closing Today: HW_1A,1B,1C
Closing next Wed: HW_2A,2B,2C
Read 5.3, 5.4 and 5.5 of the book.

5.3 The Fundamental Theorem of Calculus (FTOC)

Motivational Entry Task:
Consider the function *f(t) = 3t*.
Draw the graph and use area formulas you know, to compute:

1. $\int_0^1 f(t)dt$

2. $\int_0^{10} f(t)dt$

3. $g(x) = \int_0^x f(t)dt$

Any observations?

What changes if the lower bound a number other than 0? *For example*, let’s try 4?

4. $h(x) = \int_4^x f(t)dt$

Fundamental Theorem of Calculus (Part 1): *Areas under graphs are antiderivatives!*

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

That is, for any constant a, the "accumulated signed area" formula

$$F(x) = \int_a^x f(t)dt$$

is an antiderivative of $f(x)$.

Motivation

Assume a car is traveling at a constant $v(t) = 20$ miles/hr.

What do the following represent?

1. $\int_{2}^{5} v(t)dt$

2. $f(x) = \int_{2}^{x} v(t)dt$

3. $g(x) = \int_{0}^{x} v(t)dt$

Observations?

Mechanically using FTOC (Part 1)

Compute the **derivatives** of the following functions:

1. $g(x) = \int_{3}^{x} \cos(t)\, dt$

2. $h(x) = \int_{x}^{-2} te^{t} dt$

3. $f(x) = \int_{0}^{x^3} t + \sin(t)\, dt$

4. $k(x) = \int_{1+x^2}^{x^3} \sqrt{2+t}\, dt$

General form of FTOC (Part 1):

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\,dt\right) = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)$$

Fundamental Theorem of Calculus
(Part 2):
If F(x) <u>any</u> antiderivative of f(x),

$$\int_a^b f(x)dx = F(b) - F(a)$$

Mechanically using FTOC (Part 2)

Evaluate

1. $\int_0^1 x^3\, dx$

2. $\int_0^\pi \sin(t)\, dt$

3. $$\int_{1}^{4} \frac{1}{w}\, dw$$

5. $$\int_{1}^{2} \frac{3}{x^2}\, dx$$

4. $$\int_{0}^{5} e^x\, dx$$

6. $$\int_{1}^{4} \sqrt{x}\, dx$$

7. $\int_0^1 \frac{1}{1+x^2}\, dx$

8. $\int_0^{\pi/3} \sec(x)\tan(x)\, dx$